CS-111: Written Assignment 3

Submission instructions:

Submit your answers to the following questions in a single pdf file on Canvas & Gradescope. Your work is due by **11:59 p.m. on Wednesday, the 5th of June**.

Questions:

1) Consider a JPEG compression pipeline using 4x4 image blocks instead of 8x8. The coefficients generated by performing a DCT on a 5x5 block of luminance values are given by: **[5+5+4+4=18]**

140	20	8	5
23	18	7	8
15	10	6	2
9	5	1	2

The quantization table you are given is:

5	12	16	30
12	14	20	30
14	20	32	35
20	25	29	40

a) Generate the values of the image after quantization.

$$\hat{T}(r,c) = round\left[\frac{T(r,c)}{Z(r,c)}\right]$$

where $T(r,c)$ is the coefficients after doing DFT, $Z(r,c)$ is the quantization table values, and $\hat{T}(r,c)$ is the quantized value. The quantized values are:

28	**2**	1	**0**
2	**1**	**0**	**0**
1	**1**	**0**	**0**
0	**0**	**0**	**0**

b) This quantized table is then transmitted by linearly ordering the 4x4 matrix into a 1-D array of size 16. In what order will the quantized table be transmitted and why? Write out the linearly ordered sequence.

Zig-zag order: **28, 2, 2, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0**

c) What will be the run length encoding of this 16-element one-dimensional array?
28, (2,2), (1,3), (0,2), 1, (0,7)

d) When multiple such spatially neighboring 4x4 blocks are transmitted, what kind of additional compression schemes can be applied? Justify your answers.
DPCM, Huffman coding

2) Consider the text "*No worry and be merry*". **[2+5+10+3=20]**

a) How many characters (including spaces) does this text have?

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b) What is the probability of occurrence of each character?

n	o	w	r	y	a	d	b	e	m	space
2/21	2/21	1/21	4/21	2/21	1/21	1/21	1/21	2/21	1/21	4/21

c) Using these probabilities, construct the Huffman coding table for transmitting this text.

The Huffman table is not unique, and following is one way to construct it.

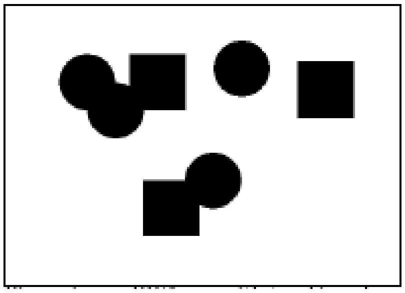
```
             0
a: 1/21----
             1 |              0
b: 1/21------(ab): 2/21--------
             0                 |
d: 1/21 ----                   |
            1    |          1  |                 0
m: 1/21-----(dm): 2/21 --------------(abdm):4/21-------
             0                                  |
o: 2/21------                                    |
             1   |               1               |                0
y: 2/21---------- (oy): 4/21------------------------------------ (abdmoy):8/21------
              0                                                          |
w: 1/21------                                                             |
              1  |              0                                         |
e: 2/21---------(we): 3/21 -----------                                    |
                             1        |         0                         |
n: 2/21-------------------------------(new): 5/12 -----                   |
               0                                  |                       |
r: 4/21----------                                 |                       |
               1     |                   1        |                 1     |
space: 4/21-------------(Sr):8/21---------------------(Srnew):13/21----------- (ALL)
```

n	o	w	r	y	a	d	b	e	m	space
101	010	1000	110	011	0000	0010	0001	1001	0011	111

d) What will be the final code that will be transmitted using this table for this text?

1010101111000010110110011111000010100101110001100111100111001110110011
(Make sure the final code matches with the Huffman Tree you build in c))

3) Shown below is a binary image with some disks and squares, some of which overlap. For each question provide an algorithm that uses morphological and logical operations to achieve the tasks mentioned. The answers may be in the form of pseudocode with a block diagram. Assume all disks are the same size and all squares are the same size. **[3+3+4=10]**



a) How many disks are there in this image?

Step-1: Erode the original image with a disk structuring element of the same size as the disk in the image.
Any disk reduces to a single pixel.

If square side length is larger than disk diameter, all squares become a cluster of connected pixels. Then, the number of single pixels gives the number of disks.

If square side length is smaller than disk diameter, all squares vanish. The number of single pixels gives the number of disks.

If square side length is the same as the disk diameter, any square also reduces to a single pixel. Then go to **Step-2**.

Step-2: Erode the original image with a square structuring element of the same size as the square in the image.
Any square reduces to a single pixel and all disks vanish.
Let **A** denote the result image from Step-1 and **B** denote the result image from Step-2. **B-A** gives you a set of isolated single pixel elements. The number of single pixels gives you the number of disks.

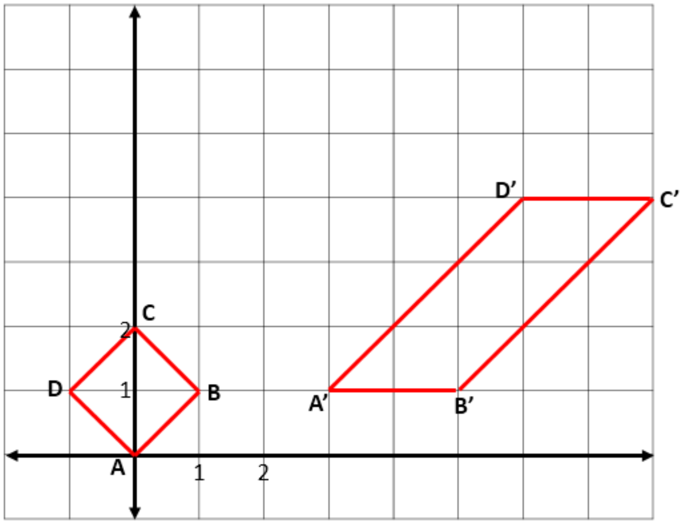
b) How many pixels form the boundary of a disk or a square?
According to (a), by eroding you can get two different images (denoted by **S** and **D**) with isolated single pixels. In **S**, each pixel corresponds to a square in the original image while in **D** each pixel corresponds to a disk in the original image.

Step-1: Dilating S/D with square/disk of the same size as the square/disk in the original image
This will create image of squares/disks only. (Squares and disks are well separated into different images).

Step-2: Extract the boundary of square/disk in S/D
This can be done by eroding with a suitable structuring element first, then performing set difference. After extracting the boundary, you can count the number of pixels forming the boundary.

c) Which pixels lie in the overlap region of two different objects?
Using the result images of Step-1 in (b), the pixels that are black in both images lie in the overlap region.

4) Consider the figure below. A sequence of transformations must be performed to transform the quadrilateral *ABCD* to the new quadrilateral *A′B′C′D′* **[5+10+5=20]**.



a) List the transformations (choose from rotation, translation, scale, shear) that must be applied sequentially, in the correct order, to perform this transformation. Note: you must list the transformations in the order that they must be applied.
There can be multiple such combinations. One such combination is:

Translate center of diamond to origin.
Rotate 45°clockwise about the origin.
Scale along x and y-axes.
Shear along x-axis .
***Translate to final position*.**

b) Provide the transformation matrices for the transformations you listed in (a).
For the transformations in (a), the matrices are:

$T_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ **Translate center of diamond to origin.**

$R = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Rotate 45° clockwise about the origin.**

$S = \begin{bmatrix} \frac{2}{\sqrt{2}} & 0 & 0 \\ 0 & \frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Scale along the x- and y-axes.**

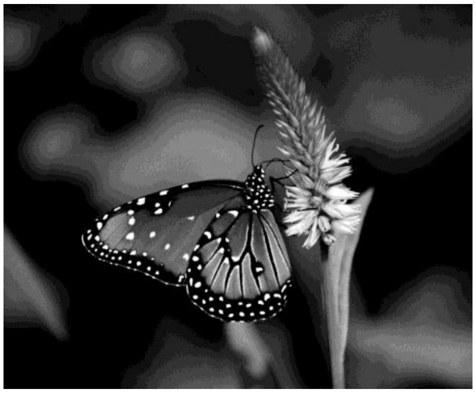
$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Shear along x-axis.**

$T_2 = \begin{bmatrix} 1 & 0 & 5.5 \\ 0 & 1 & 2.5 \\ 0 & 0 & 1 \end{bmatrix}$ **Translate to final position.**

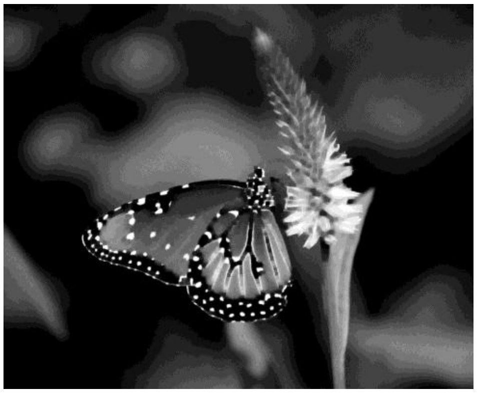
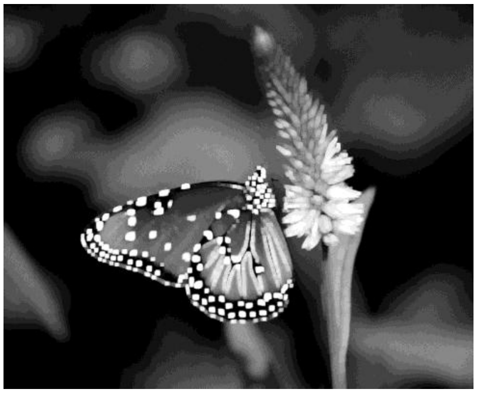
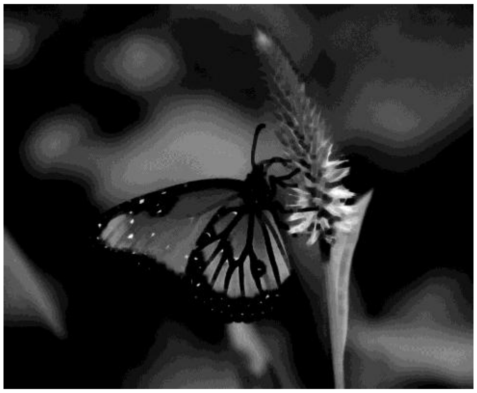
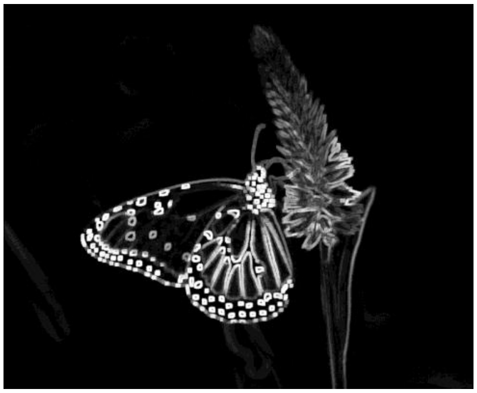
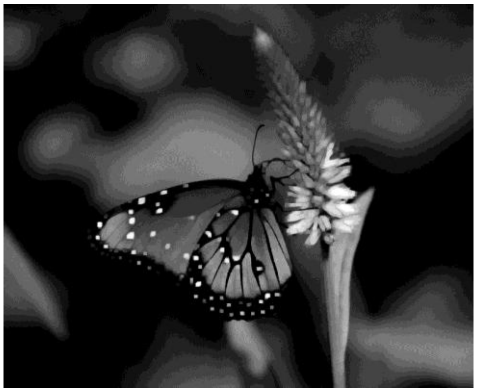
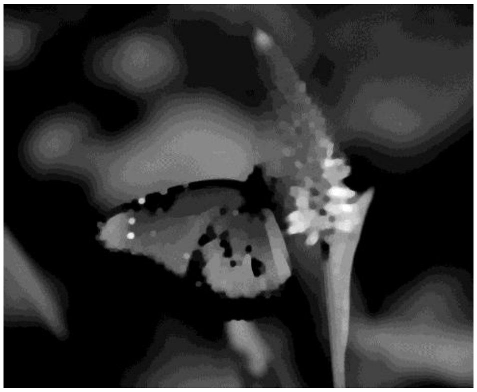
c) Compute the single transformation matrix that performs the entire operation.

Final transformation matrix = $T_2 \times H \times S \times R \times T_1 = \begin{bmatrix} -0.5 & 2.5 & 3 \\ -1.5 & 1.5 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

5) For each of the images in the table, provide the morphological operation that was applied on the input image to get the corresponding output **[2+2+2+2+2=12]**.



Input image

CLOSING  **(a)**	**DILATION**  **(b)**
EROSION  **(c)**	**GRADIENT**  **(d)**
OPENING  **(e)**	**SMOOTHING**  **(f)**

6) Consider a device with (R, G, B) chromaticity coordinates of (0.6, 0.3), (0.4, 0.5) and (0.2, 0.1) respectively. The maximum intensities for the R, G and B channels are 100, 400, and 300 respectively **[3+2+3+9+3=20]**.

a) What is the white point of this device?

The RGB channel intensities are in the following ratio: **(1/8, 4/8, 3/8)**.
Therefore, the white point of the device is at:
(0.6, 0.3) x 1/8 + (0.4, 0.5) x 4/8 + (0.2, 0.1) x 3/8 = (0.35, 0.325).

b) What is the intensity of the white of this device? **300+400+100=800**

c) What is the luminance of each of the R, G and B channel?

The intensity **I = X+Y+Z**. In the case of this device, it is 800.
The luminance of each channel is the Y coordinate.
We know that **(x, y) = (X/I, Y/I)**.
So, the luminance of each of the RGB channels is:

$$\mathbf{Y_r = 100 \times y_r = 30}$$
$$\mathbf{Y_g = 400 \times y_g = 200}$$
$$\mathbf{Y_b = 300 \times y_b = 30}$$

d) Find the color matrix for this device.

To find the color matrix, we must calculate the X, Y and Z values for each of the RGB channels. We can compute that since:

(x, y, z) = (X/I, Y/I, Z/I), where **z = 1-x-y**.

The XYZ values for each channel are computed in the table below:

	R	G	B
X	$100 \times x_r = 60$	$400 \times x_g = 160$	$300 \times x_b = 60$
Y	$100 \times y_r = 30$	$400 \times y_g = 200$	$300 \times y_b = 30$
Z	$100 \times (1-x_r-y_r) = 10$	$400 \times (1-x_g-y_g) = 40$	$300 \times (1-x_b-y_b) = 210$

The color matrix is:

$$\begin{bmatrix} 60 & 160 & 60 \\ 30 & 200 & 30 \\ 10 & 40 & 210 \end{bmatrix}$$

e) What is the XYZ value of the color generated by the input (0.2, 0.7, 0.1) in this device?

Multiplying the color matrix with the RGB input gives us the XYZ values:
(130, 149, 51).